# CBCS SCHEME

USN | | | | | | | | | | |

17MAT31

## Third Semester B.E. Degree Examination, Jan./Feb. 2023
## Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***

### Module-1

1 a. Find the Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ **(08 Marks)**

b. Find the half-range cosine series of $f(x) = \sin x$ in the interval $(0, \pi)$. **(06 Marks)**

c. Obtain the Fourier series up to first harmonic for the following data: **(06 Marks)**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| f(x) | 9 | 18 | 24 | 28 | 26 | 20 | 9 |

**OR**

2 a. Find the constant term and the first two harmonics in the Fourier series for f(x) given by the following table:

| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| f(x) | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

**(08 Marks)**

b. Obtain the Fourier series of the function $f(x) = |x|$ in $(-\ell, \ell)$. **(06 Marks)**

c. Find the half-range sine series for the function $f(x) = \ell x - x^2$ in $(0, \ell)$. **(06 Marks)**

### Module-2

3 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}, \text{ hence deduce that } \int_0^\infty \frac{(\sin x - x\cos x)}{x^3}\,dx = \frac{\pi}{4}.$$ **(08 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence show that $\int_0^\infty \frac{(x \sin mx)}{1 + x^2}\,dx = \frac{\pi e^{-m}}{2}$, $m > 0$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{3z^2 + z}{(5z - 1)(5z + 2)}$. **(06 Marks)**

**OR**

4 a. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(06 Marks)**

b. Find the Z – transform of (i) $\cosh n\theta$ (ii) $n^2$ **(06 Marks)**

c. Solve $u_{n+2} + 3u_{n+1} + 2u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$ by using Z-transforms. **(08 Marks)**

## Module-3

5 a. Using Regula-Falsi method, find a real root (correct to three decimal places) of the equation $\cos x = 3x - 1$ that lies between 0.5 and 1. (Here x is in radians) **(08 Marks)**

b. Fit a straight line of the form y = ax + b to the following data:

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 0 | 5 | 10 | 15 | 20 | 25 |
| y | 12 | 15 | 17 | 22 | 24 | 30 |

**(06 Marks)**

c. Calculate coefficient of correlation for the data below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| x | 105 | 104 | 102 | 101 | 100 | 99 | 98 | 96 | 93 | 92 |
| y | 101 | 103 | 100 | 98 | 95 | 96 | 104 | 92 | 97 | 94 |

**(06 Marks)**

**OR**

6 a. If $\theta$ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. Indicate the significance of the cases when r = 0 and r = ±1. **(08 Marks)**

b. Using Newton-Raphson method, find the real root of the equation $x\sin x + \cos x = 0$ near $x = \pi$, upto four decimal places. (Here x is in radians). **(06 Marks)**

c. Fit a parabola of the form $y = ax^2 + bx + c$ to the following data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

**(06 Marks)**

## Module-4

7 a. The population of a town is given by the table

| | | | | | |
|---|---|---|---|---|---|
| Year | 1951 | 1961 | 1971 | 1981 | 1991 |
| Population in thousands | 19.96 | 39.65 | 58.81 | 77.21 | 94.61 |

Using Newton's forward and backward interpolation formula, calculate the increase in the population from the year 1955 to 1985. **(08 Marks)**

b. Using Newton's divided difference formula, represent f(x) as a polynomial in x for the following data:

| | | | | | |
|---|---|---|---|---|---|
| x | –4 | –1 | 0 | 2 | 5 |
| f(x) | 1245 | 33 | 5 | 9 | 1335 |

Hence find f(–2). **(06 Marks)**

c. By using Simpson's 1/3 rule, evaluate $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(06 Marks)**

**OR**

8 a. Using Newton's backward interpolation formula, find the interpolating polynomial for the function given by the following table

| | | | | |
|---|---|---|---|---|
| x | 10 | 11 | 12 | 13 |
| f(x) | 21 | 23 | 27 | 33 |

Hence find f(12.5). **(08 Marks)**

b. Apply Lagrange's formula inversely to obtain a root of the equation f(x) = 0 given that f(30) = –30; f(34) = –13, f(38) = 3 and f(42) = 18. **(06 Marks)**

c. Evaluate $\int_4^{5.2} \log x\,dx$ by using Weddle's rule by taking seven ordinates. **(06 Marks)**

## Module-5

9 a. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where c is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. **(08 Marks)**

b. If $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and s is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3, evaluate $\iint_s \vec{F}.\hat{n}ds$. **(06 Marks)**

c. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

**OR**

10 a. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy - plane. **(08 Marks)**

b. Prove that geodesies of a plane are straight lines. **(06 Marks)**

c. Find the extremal of the functional $I = \int_0^{\pi/2} \left[(y')^2 - y^2 + 4y\cos x\right]dx$, given that y(0) = 0, $y\left(\frac{\pi}{2}\right) = 0$. **(06 Marks)**

* * * * *